Abstracts for TAAAG

2-4 September 2016

1 Lecture series and invited talks

Michel Raibaut (Université de Savoie). Lecture 1: Introduction to motivic integration.

Abstract. In 95', Kontsevich introduced the theory of motivic integration in order to prove that two birationally equivalent Calabi-Yau varieties have same Hodge numbers. This theory developed by Denef-Loeser and Batyrev is the analog of the p-adic integration over $C((t))$. The measurable sets are parts of arc spaces of algebraic varieties and values are "motives" meaning elements of the Grothendieck ring of varieties $K_0(Var)$. This ring is generated as a group by isomorphism classes of algebraic varieties with scissors relations. In this context, the motive of a variety X is its isomorphism class $[X]$ as element of $K_0(Var)$. In particular, this motive contains all the additive and multiplicative invariants of X, and any varieties X and Y have same such invariants as soon as the elements $[X]$ and $[Y]$ are equal in $K_0(Var)$.

Lecture 2: Motivic Milnor fibers.

Abstract. Let f be a polynomial with complex coefficients and x be a special point of the special fiber. We will recall the notion of Milnor fiber of f at x. Using arcs with origin at x, Denef and Loeser introduced a motive $S_{f,x}$ in a Grothendieck ring of varieties with action of roots of unity. This motive encodes all the additive and multiplicative invariants of the Milnor fibration of f at x and its monodromy. In particular, we can recover the monodromy zeta function or the spectrum of f at x. For that reason, it is called motivic Milnor fiber. There exists other motives wich encode invariants of global Milnor fibrations or more generally of nearby cycles of f. If we have time we will present them at the end of the talk.

Padmavathi Srinivasan (Georgia Tech). Conductors and minimal discriminants of hyperelliptic curves with rational Weierstrass points

Abstract. Conductors and minimal discriminants are two measures of degeneracy of the singular fiber in a family of hyperelliptic curves. In the case of elliptic curves, the Ogg-Saito formula shows that (the negative of) the Artin conductor equals the minimal discriminant. In the case of genus two curves, equality no longer holds in general, but the two invariants are related by an inequality. We investigate the relation between these two invariants for hyperelliptic curves of arbitrary genus.

Arnav Tripathy (Harvard). The integral Hodge conjecture and extraordinary cohomology theories (2 lectures).

Abstract. The Hodge conjecture is one of the most important problems in the cycle theory of algebraic varieties, as long as it is phrased with rational coefficients; the integral Hodge conjecture, in contrast, was disproven after only about a decade by Atiyah and Hirzebruch. I'll explain the topological viewpoint that allows you to easily guess why you should have been skeptical as developed further by Totaro, together with more recent work by Pirutka-Yagita, Kameko, Antieau, and others that allow you to apply these techniques to find easy representation-theoretic counterexamples.

Ben Williams (UBC). Lecture 1: Obstruction theory for BG.

Abstract. In the first lecture, I will discuss obstruction theory in classical homotopy, with a particular emphasis on finding explicit obstructions to maps to classifying spaces of Lie groups.

Lecture 2: Application to algebraic geometry.

Abstract. In the second lecture, I will explain how the topological methods of the first lecture may be employed to produce counterexamples in algebraic geometry, and will demonstrate with specific examples.

2 Contributed talks

Federico Buonerba (NYU). Lefschetz hyperplane theorem in Arakelov geometry.

Abstract. The Lefschetz hyperplane theorems are nowadays fundamental tools in algebraic geometry. In a joint work with M. McQuillan, we extend them to the realm of Arakelov geometry:

Let X be a projective variety defined over a number field k, $(L, ||)$ a metricised ample line bundle. Let H be the zero set of a section $s \in H^0(X, L)$, and U the metric neighborhood $|s| < 1$ in $X(C)$. Finally, denote by $H*$, resp. $X*$, the flat models of H, resp. X, over $Spec(O_k)$. Assume the following: i) $(L, ||)$ is ample in the arithmetic sense, and ii) U deformation retracts onto $H(C)$. Then the natural homotopy map $\pi_q(H^*) \to \pi_q(X^*)$ is an isomorphism if $q < dimX^* - 1$, and surjective if $q = dimX^* - 1$.

An interesting corollary is: let C/Q be a plane curve, such that $ht(C) >> 0$. Then $\pi_1(C^*)$ is trivial.

Elden Elmanto (Northwestern). The motivic sphere spectrum and étale cohomology.

Abstract. Let E be a presheaf of spectra/chain complex on some category of schemes which satisfies Nisnevich descent. Then, assuming various hypotheses, Thomason (1985), Levine (1991) and Quick (2006) prove that when is E algebraic K-theory, motivic cohomology, or algebraic cobordism respectively the mod ℓ^{ν}, "Bott-inverted" version of E also satisfies tale descent.

We outline work in progress for the motivic sphere spectrum, adapting Thomason's techniques, getting around the shortage of certain transfers in the stable motivic category by using a version of framed transfers. These results are also consistent with Gheorge's thesis, comparing the motivic sphere spectrum over $\mathbb{C}$ and the classical sphere spectrum.

Xing Gu (UIC). On the Cohomology of the Classifying Spaces of Projective Unitary Groups.

Abstract. Let $\mathbf{B}PU(n), \mathbf{B}U(n)$, and $K(\mathbb{Z}, 3)$ denote respectively the projective unitary group of rank n, the unitary group of rank n, and the Eilenberg-Maclane space with the third homotopy group being $\mathbb{Z}$. We construct a cohomological Serre spectral sequence $E_*^{*,*}$ with $E_2^{s,t} \simeq H^s(K(\mathbb{Z},3), H^t(\mathbf{B}U(n)))$ and converging to $H^*(\mathbf{B}PU(n))$. Moreover we determine all of its differentials. This enables us to calculate $H^*(\mathbf{B}PU(n))$ up to extension.

Leo Herr (CU Boulder). Deformations of modules.

Abstract. Illusie's introduction of the cotangent complex in 1971 solved and generalized previous questions about the possibility of extending modules. However, the direct local-to-global approach to the problem that existed previously suffices to solve the same problem and does not depend as much on simplicial machinery. We translate between the two approaches using a comparison between cohomology and extensions. (Joint work with Jonathan Wise, building on previous papers on the deformations of algebras).

Artur Jackson (Purdue). Some Geometry over $\mathbb{F}_1$.

Abstract. I will quickly sketch the machine of Ton and Vaqui for producing categories of schemes relative to an arbitrary cosmos. Ill state a Tannakian theorem that works in this setting, briefly sketch how to manufacture a compactification of $Spec\,\mathbb{Z}$ following Durov and Connes-Consani, and finally will mention how a proposal of Smirnov for the *abc*-conjecture can be phrased in this the language of relative schemes.

Daniel Litt (Columbia). Arithmetic restrictions on geometric monodromy

Abstract: Let X be an algebraic variety over a field k. Which representations of $\pi_1(X)$ arise from geometry, e.g. as monodromy representations on the cohomology of a family of varieties over X? We study this question by analyzing the action of $Gal(\bar{k}/k)$ on $\pi_1(X)$, where k is a finite or p-adic field. As a sample application of our techniques, we show that if A is a non-constant Abelian variety over $\mathbb{C}(t)$, such that $A[\ell]$ is split for some odd prime ℓ, then A has at least four points of bad reduction.

Patrick McFaddin (University of South Carolina). K_1-zero-cycles for some twisted Grassmannians.

Abstract. Chow groups with coefficients, as defined and studied by Markus Rost, are a reasonable analogue for singular homology (with coefficients) of algebraic varieties and have proved useful in the study of central simple algebras and Galois cohomology. Unfortunately, a general description of Chow groups (with coefficients) remains elusive, and computations of these groups are done in various cases. In this talk, I will discuss some recent work on computing these groups for certain generalized Severi-Brauer varieties associated to algebras of index 4.

Jackson Morrow (Emory). Explicit methods in arithmetic geometry.

Abstract. In Mazurs celebrated 1978 Inventiones paper, he classified the torsion subgroups which can occur in the Mordell-Weil group of an elliptic curve over $\mathbf{Q}$. His result was extended to elliptic curves over quadratic number fields by Kamienny, Kenku, and Momose, with the full classification being completed in 1992. What both of these cases have in common is that each subgroup in the classification occurs for infinitely many elliptic curves; however, this no longer holds for cubic number fields. In 2012, Najman showed that there exists a unique (up to $\overline{\mathbf{Q}}$-isomorphism) elliptic curve whose torsion subgroup over a particular cubic field is $\mathbf{Z}/21\mathbf{Z}$ by exhibiting a cubic point on the modular curve $X_1(21)$. This curve yielded the first "sporadic" example of a torsion subgroup. In this talk, we will discuss our explicit method for the analysis of cubic points on the modular curves $X_1(N)$ and $X_1(M, N)$.

Ben Obeiro (Technical University of Kenya). On minimal resolution conjecture for the ideal of in $\mathbb{P}^4$.

Abstract. In this talk, we show that the map,

$$\mathrm{H}^0\left(\mathbb{P}^4, \Omega^2_{\mathbb{P}^4}(d+2)\right) \longrightarrow \bigoplus_{i=1}^{s} \Omega_{\mathbb{P}^4}(d+2)|_{P_i},$$

is of maximal rank using the method of Horace. This implies that the number of generators of degree $d+2$ of I_S, the ideal of $\{P_1, \cdots, P_s\} \subset \mathbb{P}^4$ in general position is $\left|h^0(\mathbb{P}^4, \Omega^2_{\mathbb{P}^4}(d+1)) - 6s\right|$. That is, the betti numbers a_1 and b_1 of the minimal free resolution of I_S satisfy $a_1 b_1 = 0$.

Jasmin Omanovic (UWO). Involutions and the classification of quadratic forms.

Abstract. The study of quadratic forms may seem far removed from the study of (central) simple algebras, and in general, this is indeed the case. However, if the central simple algebra carries an involution (such as matrix algebras and quaternion algebras) then we have a different story. In this talk, we will briefly outline the relationship between algebras with an involution and quadratic forms (assuming characteristic is not 2). In particular, we will discuss the relevance of classification results for quadratic forms in I^3 (with low dimension) to the structure theory of central simple algebras with orthogonal involution.